

Calculating Linkage Disequilibrium

SNP1
 ACT**G**GTAT.....GATCA**A**CCAG
 ACT**C**GTAT.....GATCA**A**CCAG
 ACT**C**GTAT.....GATCA**T**CCAG

Alleles	SNP1	SNP2
Allele 1	G	A
Allele 2	C	T

Step 1) Calculate allele frequencies

SNP1		SNP1	
Allele	Frequency	Allele	Frequency
G	p1	A	q1
C	p2	T	q2

Step 2) Calculate haplotype frequencies

Allele	Frequency	Allele	Frequency
GA	p11	GT	q12
CA	p21	CT	q22

Step 3) Check for linkage equilibrium

Haplotype Frequency	Product of Allele Frequency
p11	p1*q1
p12	p1*q2
p21	p2*q1
p22	p2*q2

Step 4) Calculate linkage disequilibrium

		SNP 2		
		1	2	
SNP 1	1	p1q1 + D	p1q2 - D	p1
	2	p2q1 - D	p2q2 + D	p2
		q1	q2	1

Solve for D

...

$$\begin{aligned}
 p_{11} p_{22} &= (p_1 q_1 + D)(p_2 q_2 + D) = p_1 q_1 p_2 q_2 + p_1 q_1 D + p_2 q_2 D + D^2 \\
 p_{12} p_{21} &= (p_1 q_2 - D)(p_2 q_1 - D) = p_1 q_1 p_2 q_2 - p_2 q_1 D - p_1 q_2 D + D^2 \\
 (p_{11} p_{22} + p_1 q_1 D + p_2 q_2 D + D^2) &- (p_{12} p_{21} - p_2 q_1 D - p_1 q_2 D + D^2)
 \end{aligned}$$

$$D = p_{11} p_{22} - p_{12} p_{21}$$

Step 5) Calculate Pearson coef.

$$r^2 = [D / (p_1 p_2 q_1 q_2)^{1/2}]^2$$

Step 6) Sum r^2 for all neighbor SNPs

LD window = 1 centimorgan

$$\text{LDscore} = \sum r^2$$